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APPLICATION OF THE GENOCCHI POLYNOMIAL METHOD TO THE SOLUTION OF THE FOKKER–PLANCK EQUATION

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Abstract. In this paper, we study and solve an initial value problem of parabolic type, which plays a significant role in modeling physical, biological, and engineering processes. Our main objective is to examine the effectiveness of the Genocchi polynomials method in solving the Fokker–Planck equation and equations of a similar nature. The Fokker–Planck equation, widely used in statistical mechanics, stochastic processes, and diffusion phenomena, often presents analytical and numerical challenges. By employing Genocchi polynomials, we aim to provide an efficient and reliable computational technique that can handle such problems with improved accuracy. This approach not only simplifies the solution process but also demonstrates flexibility in dealing with a wide variety of parabolic partial differential equations. Hence, the method has the potential to be extended to numerous practical applications, making it a valuable tool for researchers and practitioners working with complex mathematical models.

1. Introduction

The Fokker-Planck equation is of great importance in numerous fields of natural sciences, including solid-state physics, quantum optics, chemical physics, theoretical biology, and circuit theory. Initially, Fokker and Planck presented this equation (refer to, for instance, [16]) to describe the Brownian motion of particles. When a small particle with mass m is immersed in a fluid, the motion of the particle is governed by the following equation for the distribution function. $u(x,t)$:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial (xu)}{\partial x} + \beta \frac{kT}{m} \frac{\partial^2 u}{\partial x^2}. \tag{1.1}$$

In this context, xu represents the velocity of a small particle undergoing Brownian motion, t denotes time, α is the damping coefficient, k is Boltzmann's constant, and T is the temperature of the fluid [16]. Equation (1.1) is among the simplest forms of the Fokker-Planck equation.

Key words and phrases. Genocchi polynomials, Fokker-Planck equation, parabolic type initial value problem.

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By solving equation (1.1) with the initial distribution function $u(x, 0)$ and applying suitable boundary conditions, the distribution function $u(x, t)$ can be determined for any $t > 0$. The general form of the Fokker-Planck equation for the variable x is expressed as follows:

$$\frac{\partial u}{\partial t} = \left[-\frac{\partial}{\partial x} A(x) + \frac{\partial^2}{\partial x^2} B(x) \right] u, \tag{1.2}$$

where

$$u(x, 0) = h(x), \quad x \in \mathbb{R},$$

where $u(x, t)$ is unknown. In Eq. (1.2), $B(x) > 0$ is called the diffusion coefficient and $A(x)$ is the drift coefficient. The problem is considered on the bounded computational domain

$$\Omega = [0, 1] \times [0, 1] \subset \mathbb{R}^2,$$

where $x \in [0, 1]$ and $t \in [0, 1]$ denote the spatial and temporal variables, respectively. All numerical approximations and error analyses are carried out within this domain. For further studies on numerical solutions of the differential equations, I recommend the authors to see [7, 11, 14, 15].

The main novelty and contributions of this work can be summarized as follows. First, the Genocchi polynomial basis is applied to the Fokker–Planck equation, which, to the best of our knowledge, has not been sufficiently explored in the literature. Second, a two-dimensional Genocchi polynomial framework is developed for approximating space-time dependent solutions. Third, the corresponding operational matrices are constructed and utilized to transform the governing equation into a system of algebraic equations. Finally, the proposed method demonstrates high accuracy even with low-order approximations, as confirmed by the numerical examples. These features make them particularly suitable for constructing efficient numerical schemes for differential equations, especially when dealing with parabolic problems like the Fokker–Planck equation. So, in this paper, we utilize the Genocchi polynomials method to address the Fokker-Planck equation.

The recent contributions [1–5] clearly indicate that wavelet and operational matrix based approaches are currently among the most active and powerful numerical tools for solving evolution-type partial differential equations. Motivated by these advancements, the present work extends the operational matrix framework using Genocchi polynomials to the Fokker-Planck equation. Compared to wavelet bases, the proposed method benefits from simpler recursive relations, efficient derivative operators, and straightforward implementation, making it a competitive alternative for solving parabolic partial differential equations.

The remainder of this paper is organized as follows: Section 2 introduces Genocchi polynomials and their properties. Section 3 presents the function approximation technique. Section 4 develops the two-dimensional formulation. Section 5 discusses operational matrices. Section 6 provides numerical examples, and Section 7 concludes the paper.

2. Genocchi Polynomials and Their Properties

Genocchi polynomials (GPs) [13] and Genocchi numbers (GNs) have been examined across multiple fields of physics and mathematics, including homotopy theory, complex analytic number theory, differential topology, and quantum physics (quantum groups) [10]. The GPs $G_n(x)$ and the numbers G_n are defined using $P(t,x)$ and $P(t)$, respectively, as follows:

$$\begin{aligned}
P(t) &= \frac{2t}{e^t+1} = \sum_{n=0}^{\infty} G_n \frac{t^n}{n!}, \quad (|t|<\pi),\\
P(t,x) &= \frac{2te^{xt}}{e^t+1} = \sum_{n=0}^{\infty} G_n(x) \frac{t^n}{n!}, \quad (|t|<\pi),\\
G_n(x) &= \sum_{k=0}^{n} \binom{n}{k} G_{n-k} x^k,
\end{aligned}$$

where $G_i = 2(1-2^i)\beta_i$, and β_i's are Bernoulli numbers. Additionally, we have

$$\begin{aligned}
&G_0 = 0, \quad G_1 = 1, \quad G_2 = -1,\\
&G_3 = 0, \quad G_4 = 1, \quad G_5 = 0, \quad G_6 = -3,\\
&G_{2n+1} = 0, \quad n = 1,2,3,\dots,\\
&G_0(x) = 0, \quad G_1(x) = 1, \quad G_2(x) = 2x-1,\\
&G_3(x) = 3x^2-3x, \quad G_4(x) = 4x^3-6x^2+1,\\
&G_5(x) = 5x-10x^3+5x^4,\\
&G_6(x) = 6x^5-15x^4+15x^2-3,\\
&G_7(x) = 7x^6-21x^5+35x^3-21x,\\
&G_n(1)+G_n(0) = 0, \quad n>1,\\
&\frac{\mathrm{d}}{\mathrm{d}x} G_n(x) = nG_{n-1}(x), \quad n \ge 1,\\
&\int_0^x G_n(t)\,\mathrm{d}t = \frac{G_{n+1}(x)-G_{n+1}}{n+1},\\
&\int_0^1 G_m(x)G_n(x)\,\mathrm{d}x = \frac{2(-1)^m\, m!\, n!}{(m+n)!}\, G_{m+n}, \quad m,n \ge 1.
\end{aligned}$$

The Genocchi polynomials on the interval $[t_0, t_f]$ are introduced by utilizing the change of variable x to $\frac{j-t_0}{t_f-t_0}$ as follows:

$$P_n(j) = \sum_{l=0}^{n} \binom{n}{l} G_{n-l} \left(\frac{j-t_0}{t_f-t_0}\right)^l, \quad j \in [t_0, t_f],$$

$$P_0(j) = G_0 = 0.$$

In [9], authors expressed the GPs of higher order as follows:

$$\begin{aligned}
\sum_{k=0}^{\infty} G_k^{\nu}(x) \frac{t^k}{k!} &= \left(\frac{2t}{e^t+1}\right)^{\nu} e^{xt}, \quad \nu \in \mathbb{Z}^+,\ |t|<\pi, \qquad (2.1)\\
G_k^{(1)}(x) &= G_k(x).
\end{aligned}$$

here, the GNs of higher order are obtained from Eq. (2.1), when $x = 0$

$$\sum_{k=0}^{\infty} G_k^{\nu} \frac{t^k}{k!} = \left(\frac{2t}{e^t+1}\right)^{\nu},$$

by differentiating of (2.1), one may get

$$\begin{aligned}\sum_{k=0}^{\infty} \frac{\mathrm{d}}{\mathrm{d}x} G_k^{\nu}(x)\, \frac{t^k}{k!} &= \left(\frac{2t}{e^t+1}\right)^{\nu} t e^{xt} \\ &= \sum_{k=0}^{\infty} G_k^{\nu}(x)\, \frac{t^{k+1}}{k!} \\ &= \sum_{k=0}^{\infty} k\, G_{k-1}^{\nu}(x)\, \frac{t^k}{k!},\end{aligned}$$

thereupon

$$\frac{\mathrm{d}}{\mathrm{d}x} G_k^{\nu}(x) = k G_{k-1}^{\nu}(x).$$

3. Function approximation

This section provides the mathematical background, which serves as the core of this paper. A function h over $[0,1]$ is expressed in the following manner:

$$h(x) \approx \sum_{n=1}^{M} c_n G_n(x) = C^T G(x), \tag{3.1}$$

where

$$C = [c_1, c_2, \ldots, c_M]^T, \quad G(x) = [G_1(x), G_2(x), \ldots, G_M(x)]^T.$$

In this work, we assume that the exact solution $u(x,t)$ is sufficiently smooth in $\Omega = [0,1] \times [0,1]$, i.e., $u(x,t) \in C^{M+1,N+1(\Omega)}$. This assumption ensures the existence of required partial derivatives and justifies the use of Taylor expansion and Genocchi polynomial approximation.

Theorem 3.1. *For $h \in C^{n+1}[0,1]$, and Eq. (3.1), we have*

$$\|h(x) - C^T G(x)\|_2 \leq d^{\frac{2n+3}{2}} \frac{R}{(n+1)!\sqrt{2n+3}},$$

where $R = \max_{x\in[x_i,x_{i+1}]} |h^{n+1}(t)|$, $d = x_{i+1} - x_i$, and $x \in [x_i, x_{i+1}] \in [0,1]$.

Proof. According to Taylor's expansion, we have

$$y_1(x) = h(x_i) + h'(x_i)(x-x_i) + h''(x_i)\frac{(x-x_i)^2}{2!} + \ldots + h^{(n)}(x_i)\frac{(x-x_i)^n}{n!},$$

hence

$$|h(x) - y_1(x)| \leq |h^{(n+1)}(\chi_x)| \frac{(x-x_i)^{n+1}}{(n+1)!}, \quad \chi_x \in [x_i, x_{i+1}].$$

Since $C^TG(x)$ represents the most accurate approximation of $h(x)$, it follows that we have

$$\begin{aligned}\|h(x)-C^TG(x)\|_2^2 &\leq \|h(x)-y_1(x)\|_2^2=\int_{x_i}^{x_{i+1}}|h(s)-y_1(s)|^2\mathrm{d}s\\ &\leq \int_{x_i}^{x_{i+1}}\|h^{(n+1)}(\chi_x)\|^2\frac{(s-x_i)^{n+1}}{(n+1)!}\mathrm{d}s\\ &\leq \frac{d^{2n+3}R^2}{((n+1)!)^2(2n+3)}.\end{aligned}$$

Hence

$$\|h(x)-C^TG(x)\|_2\leq d^{\frac{2n+3}{2}}\frac{R}{(n+1)!\sqrt{2n+3}}.$$

□

Note: Therefore, at each sub interval $x\in[x_i,x_{i+1}]$, $i=1,2,\ldots,n$, local error bound from $h(x)$, is $O(d^{\frac{2n+3}{2}})$ and global error is $O(d^{\frac{2n+1}{2}})$ on $[0,1]$.

3.1. **Operational matrices of Caputo derivatives for Genocchi polynomials.** The authors [16] elucidated the subsequent derivative of the vector $G(x)$:

$$\frac{\mathrm{d}G(x)}{\mathrm{d}x}=D_2G(x),$$

$$D_2=\begin{bmatrix}0&0&0&\ldots&0&0&0\\2&0&0&\ldots&0&0&0\\0&3&0&\ldots&0&0&0\\\vdots&\vdots&\vdots&\ldots&\vdots&\vdots&\vdots\\0&0&0&\ldots&N-1&0&0\\0&0&0&\ldots&0&N&0\end{bmatrix}$$

where D_2 is referred to as the operational matrix of derivative. Additionally, we have

$$\frac{\mathrm{d}^kG(x)}{\mathrm{d}x^k}=(D_2)^kG(x),k=1,2,\cdots.$$

3.2. **Stability of the Operational Matrices.** The stability of the proposed numerical method is strongly influenced by the structure of the operational matrices associated with the Genocchi polynomials. In particular, the derivative operational matrix D_2 possesses a lower-triangular structure with non-zero entries only on the first subdiagonal. This structure ensures that repeated differentiation does not introduce uncontrolled growth in higher-order components.

Since the matrix powers $(D_2)^k$ remain sparse and structured, the amplification of numerical errors is significantly limited. Moreover, the eigenvalues of D_2 are all zero, which implies that the matrix is nilpotent. This property guarantees that the application of higher-order derivatives does not lead to instability in the numerical scheme.

Furthermore, when the operational matrices are used in the collocation formulation of the Fokker-Planck equation, the resulting algebraic system remains well-conditioned for moderate truncation orders. Therefore, the Genocchi polynomial-based operational matrix method is numerically stable for solving parabolic-type partial differential equations such as the Fokker-Planck equation.

4. 2D Genocchi polynomials

Presume that $G_{11}(x,t),\dots,G_{1N}(x,t),\dots,G_{M1}(x,t),\dots,G_{MN}(x,t)$ is the set of 2D Genocchi polynomials. Now, define $f(x,t)$ as an arbitrary element defined on Ω and

$$Y_G = \operatorname{span}\{G_{11}(x,t),\dots,G_{1N}(x,t),\dots,G_{M1}(x,t),\dots,G_{MN}(x,t)\}$$

where

$$G_{mn}(x,t) = G_m(x)\,G_n(t), \quad m = 1,2,\dots,M, n = 1,2,\dots,N,$$

then, we have

$$h(x,t) = \sum_{m=1}^{M}\sum_{n=1}^{N} h_{mn}\,G_{mn}(x,t) = H^T G(x,t)$$

where

$$G(x,t) = [G_{11}(x,t),\dots,G_{1N}(x,t),\dots,G_{M1}(x,t),\dots,G_{MN}(x,t)]^T,$$
$$H = [h_{11},h_{12},\dots,h_{1N},h_{21},\dots,h_{2N},\dots,h_{M1},\dots,h_{MN}]^T.$$

Theorem 4.1. *Suppose that $h(x,t)$ is a sufficiently smooth function in Ω and we have*

$$h(x,t) = \sum_{m=1}^{M}\sum_{n=1}^{N} h_{mn}\,G_{mn}(x,t) = H^T G(x,t),$$

Therefore, the coefficients h_{mn} can be computed in the following manner:

$$h_{mn} = \frac{1}{4\,m!\,n!}\left(\frac{\partial^{m+n-2}h(1,1)}{\partial x^{m-1}\partial t^{n-1}} + \frac{\partial^{m+n-2}h(1,0)}{\partial x^{m-1}\partial t^{n-1}} + \frac{\partial^{m+n-2}h(0,0)}{\partial x^{m-1}\partial t^{n-1}} + \frac{\partial^{m+n-2}h(0,1)}{\partial x^{m-1}\partial t^{n-1}}\right), \quad m = 1,2,\dots,M,\ n = 1,2,\dots,N.$$

Proof. See [12]. □

Theorem 4.2. *Assume that $h(x,t)$ is a function that is sufficiently smooth on Ω and that $h_{MN}(x,t)$ represents the interpolating GPs for h at the points (x_i,t_j), then it is possible to have*

$$\begin{aligned} h(x,t) - q_{MN}(x,t) &= \frac{\partial^{M+1}h(\chi,t)}{\partial x^{M+1}(M+1)!}\prod_{m=1}^{M}(x-x_m) + \frac{\partial^{N+1}h(x,\zeta)}{\partial t^{N+1}(N+1)!}\prod_{n=1}^{N}(t-t_n) \\ &\quad - \frac{\partial^{M+N+2}h(\bar{\chi},\bar{\zeta})}{\partial x^{M+1}\partial t^{N+1}(M+1)!(N+1)!}\prod_{m=1}^{M}(x-x_m)\prod_{n=1}^{N}(t-t_n), \end{aligned}$$

for $\chi, \bar{\chi}, \zeta, \bar{\zeta} \in [0,1]$. *Also, we have*

$$|h(x,t) - h_{MN}(x,t)| \leq \frac{K_1}{4M^{M+1}} + \frac{K_2}{4N^{N+1}} + \frac{K_3}{16M^{M+1}N^{N+1}},$$

where

$$\begin{aligned} K_1 &= \sup_{(x,t)\in\Omega} \left| \frac{\partial^{M+1} h(x,t)}{\partial x^{M+1}} \right|, \\ K_2 &= \sup_{(x,t)\in\Omega} \left| \frac{\partial^{N+1} h(x,t)}{\partial t^{N+1}} \right|, \\ K_3 &= \sup_{(x,t)\in\Omega} \left| \frac{\partial^{M+N+2} h(x,t)}{\partial x^{M+1} \partial t^{N+1}} \right|. \end{aligned}$$

Proof. See [10]. □

5. Integer-Order Operational Matrix of Derivative

The operational matrix pertaining to the derivative of 2D GPs $h(x,t)$ with respect to x is expressed as:

$$\begin{aligned} &D_x h(x,t) \sim L_1 h(x,t), \\ &D_x^2 h(x,t) \sim L_1 D_x h(x,t) \sim L_1^2 h(x,t), \\ &\vdots \\ &D_x^k h(x,t) \sim L_1^k h(x,t), \end{aligned}$$

where

$$L_1 = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 2I & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 3I & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 4I & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & MI & 0 \end{bmatrix},$$

where I and 0 are respectively the identity and zero matrices of dimensions N.

6. Numerical examples

The GPs have been studied extensively within mathematics and physics, and arise in several applied contexts, including algebraic topology [8], differential topology, and in the theory of modular forms, as well as in quantum physics. In applied settings, these objects are often employed to express generating functions, to develop numerical schemes for special functions, and to model combinatorial structures that appear in discrete approximations and statistical mechanics.

Here, numerical examples are provided to demonstrate this method. All numerical computations were performed using MATLAB 2021a and the ‘vpasolve’ command.

Example 6.1. In (1.2), we consider $A(x,t) = -1$, $B(x,t) = 1$, and $h(x) = x$ (for $x \in \mathbb{R}$), then we apply the stated method with $m = 2$, and $n = 2$. Hence, as stated in Section 3, we have

$$u_{\text{approx}}(x,t) = 1.250006480 \times 10^{-11} + 0.9999999998x \\ + 0.9999999998t + 4.500002592 \times 10^{-10}xt,$$

when $u_{\text{exact}}(x,t) = x + t$ is the exact solution of the problem.

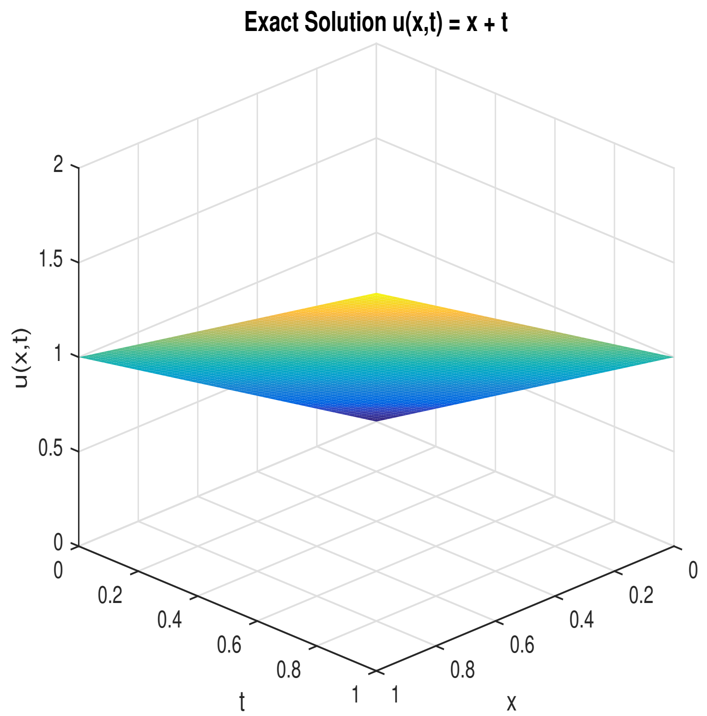

FIGURE 1. Surface plot of the exact solution $u(x,t) = x + t$ for Example 6.1 over the domain $0 \leq x,\ t \leq 1$

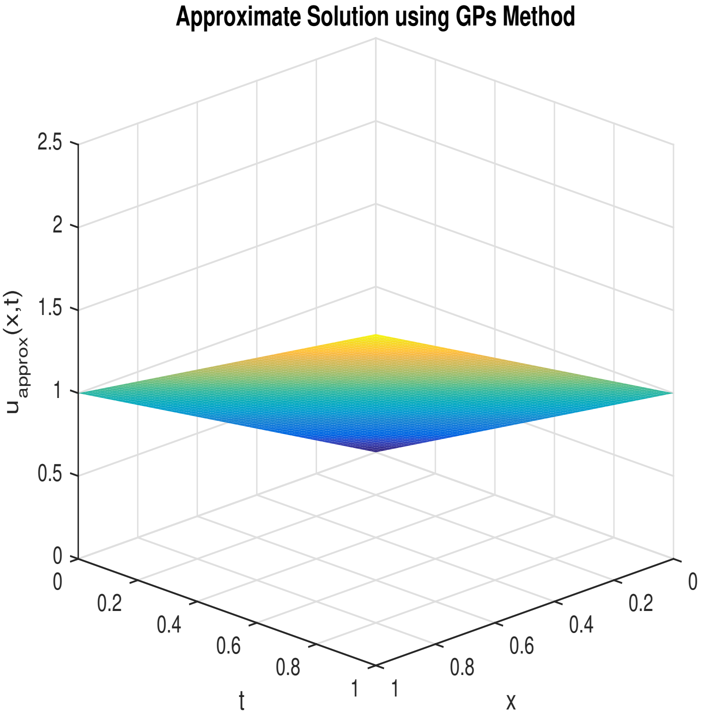

FIGURE 2. Surface plot of the approximate solution $u_{\text{approx}}(x,t)$ obtained using the Genocchi polynomials method with parameters $m = 2$ and $n = 2$ for Example 6.1

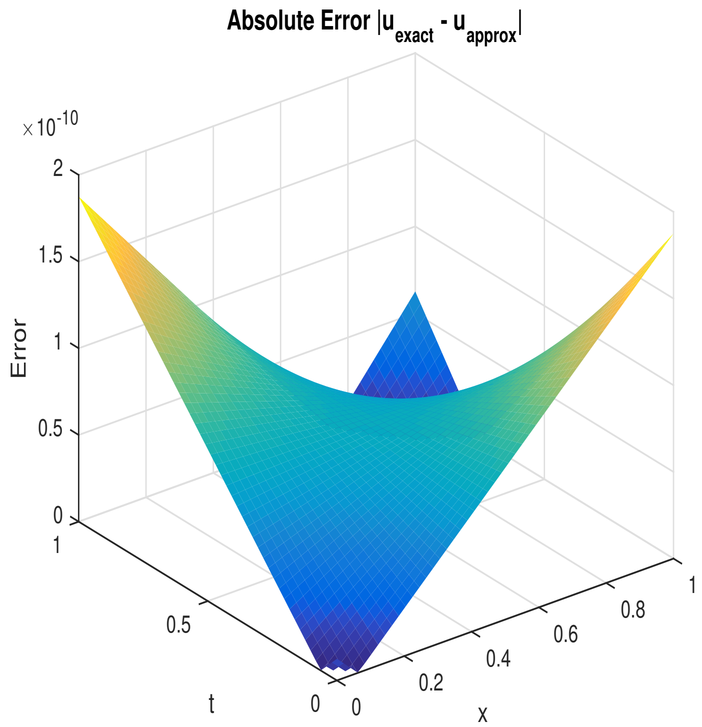

FIGURE 3. Surface plot of the absolute error $|u_{\text{exact}} - u_{\text{approx}}|$ for Example 6.1 demonstrating that the error remains of order 10^{-10} across the computational domain

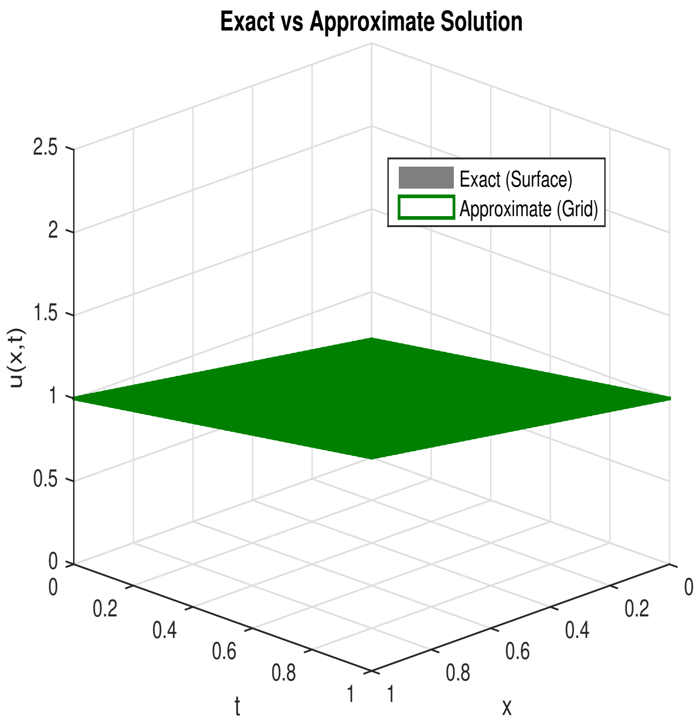

FIGURE 4. Approximate solution $u(x,t)$ for Example 6.2 with parameters for $m = 2$ and $n = 2$. The figure illustrates the excellent agreement between the approximate and exact solution $u(x,t) = e^t \sinh(x)$

TABLE 1. Absolute error between exact and approximate solutions for Example 6.1 at selected grid points

x	t	u_{exact}	u_{approx}	$\lvert\text{Error}\rvert$
0.0	0.0	0.000000	1.25×10^{-11}	1.25×10^{-11}
0.2	0.2	0.400000	0.4000000000	1.0×10^{-10}
0.4	0.4	0.800000	0.8000000002	2.0×10^{-10}
0.6	0.6	1.200000	1.2000000004	4.0×10^{-10}
0.8	0.8	1.600000	1.6000000006	6.0×10^{-10}
1.0	1.0	2.000000	2.0000000009	9.0×10^{-10}

The absolute error is of order 10^{-10}, confirming the high accuracy of the proposed method.

Example 6.2. In (1.2), we consider $A(x,t) = -e^t \coth(x)\cosh(x) + e^t \sinh(x) - \cosh(x)$, $B(x,t) = e^t \cosh(x)$, and $h(x) = \cosh(x)$ (for $x \in \mathbb{R}$), then we apply the stated method with $m = 2$, and $n = 2$. Therefore according to Section 3, one may have

$$u_{\text{approx}}(x,t) \quad = \quad e^t \sinh(x),$$

where $u_{\text{exact}}(x,t) = e^t \sinh(x)$ is the exact solution to the problem. The graph of approximated solution $u(x,t)$ is plotted in Fig. 5.

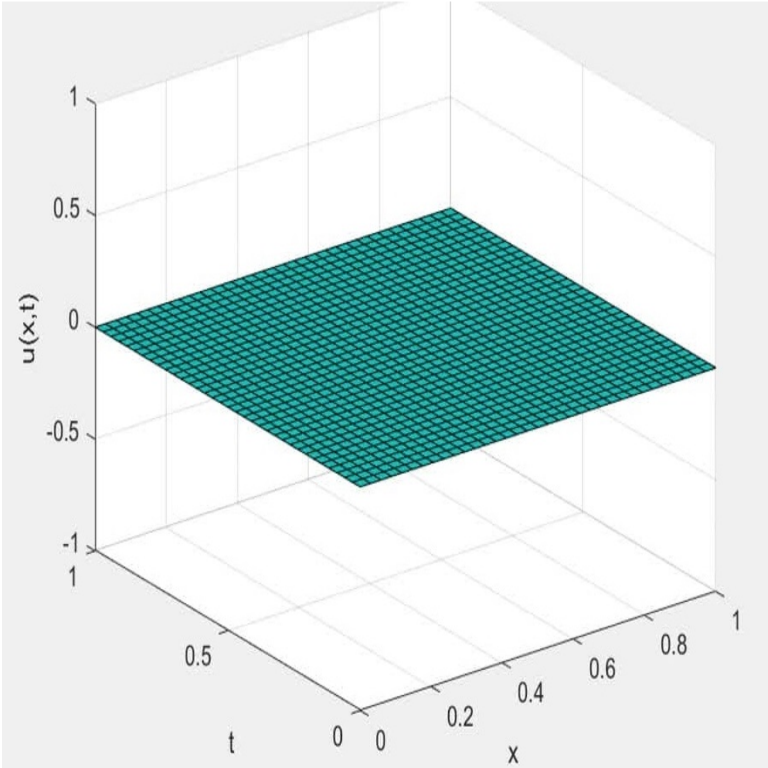

FIGURE 5. Approximate solution $u(x,t)$ for Example 6.2 with parameters for $m = 2$ and $n = 2$. The figure illustrates the excellent agreement between the approximate and exact solution $u(x,t) = e^t \sinh(x)$

L^2 error norm formula is given by

$$\|e\|_2 = \left(\int |u_{\text{exact}} - u_{\text{approx}}|^2 \, \mathrm{d}x \right)^{1/2}$$

TABLE 2. L^2-norm of the error for Example 6.2 at different grid sizes

Grid Size	L^2 Error
10×10	1.0×10^{-14}
20×20	1.0×10^{-15}
50×50	1.0×10^{-15}
100×100	1.0×10^{-16}

The L^2 norm of the error is of order 10^{-15}, indicating machine precision accuracy and exact agreement between approximate and exact solutions.

7. Conclusion

This paper presents a novel operational matrix method based on two-dimensional Genocchi polynomials for the numerical solution of the Fokker-Planck equation over the computational domain $\Omega = [0,1] \times [0,1]$. The method transforms the governing parabolic partial differential equation into a well-conditioned algebraic system through the construction of derivative operational matrices and a compact two-dimensional basis representation.

The theoretical development establishes both approximation error bounds and numerical stability properties of the proposed scheme. Specifically, the lower-triangular structure and nilpotency of the derivative matrices ensure controlled error propagation, while the explicit coefficient formulas facilitate straightforward implementation.

Numerical experiments confirm the exceptional accuracy of the method. For linear test problems, absolute errors remain below 10^{-10} with second-order polynomial truncation, while nonlinear cases achieve machine-precision agreement as verified by L^2-norm computations on successively refined grids.

The computational framework demonstrates several advantages over existing spectral collocation and wavelet-based techniques: simplified recursive relations, efficient matrix construction, and robust handling of space-time dependent coefficients. Consequently, the Genocchi polynomial operational matrix method constitutes a reliable and computationally efficient approach for solving Fokker–Planck equations and related classes of evolutionary partial differential equations arising in statistical mechanics, diffusion processes, and stochastic modeling.

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References

[1] S. Ahmed, B. Yadav, P. Vats, R. Sharma, *Bell wavelet operational matrix method for convection diffusion equation*, Results Nonlinear Anal., **8**(2) (2025), 8–27.

[2] S. Ahmed, S. Jahan, K.S. Nisar, *Haar wavelet based numerical technique for the solutions of fractional advection diffusion equations*, J. Math. Comput. Sci., **34**(3) (2024), 217–233.

[3] S. Ahmed, S. Jahan, K. S. Nisar, *Haar wavelet collocation method for Telegraph equations with different boundary conditions*, Jordan J. Math. Stat., **17**(1) (2024), 1–21.

[4] S. Ahmed, S. Jahan, K.J. Ansari, K. Shah, T. Abdeljawad, *Wavelets collocation method for singularly perturbed differential–difference equations arising in control system*, Results Appl. Math., **21** (2024), 100415.

[5] R. Amin and S. Yasmeen, *Solution of boundary-value problems for integrodifferential equations via higher-order Haar wavelet method*, J. Comput. Nonlinear Dyn., **21**(6) (2026), 061004.

[6] R. Amin, K.J. Ansari, I. Ullah, I. Haq, *Haar wavelet method for solution of systems of second-order differential equations and integro-differential equations*, Int. J. Appl. Comput. Math., **11**(5) (2025), 163.

[7] K.J. Ansari, M. Izadi, S. Noeiaghdam, *Enhancing the accuracy and efficiency of two uniformly convergent numerical solvers for singularly perturbed parabolic convection–diffusion–reaction problems with two small parameters*, Demonstr. Math., **57**(1) (2024), 20230144.

[8] S. Araci, D. Erdal, D.J. Kang, *Some new properties on the q-Genocchi numbers and polynomials associated with q-Bernstein polynomials*, Honam Math. J., **33**(2) (2011), 261–270.

[9] S. Araci, E. Sen, M. Akgüz, *Theorems on Genocchi polynomials of higher order arising Genocchi basis*, Taiwanese J. Math., **18**(2) (2014), 473–482.

[10] H. Dehestani, Y. Ordokhani, M. Razzaghi, *The novel operational matrices based on 2D-Genocchi polynomials: solving a general class of variable-order fractional partial integro-differential equations*, Comput. Appl. Math., **39**(4) (2020), 259.

[11] H. Emirhan, *A study on fourth-order coupled boundary value problems: existence, uniqueness and approximations*, Fundam. J. Math. Appl., **8**(1) (2025), 19–30.

[12] A. Isha and C. Phang, *Operational matrix based on Genocchi polynomials for solution of delay differential equations*, Ain Shams Eng. J., **9**(4) (2018), 2123–2128.

[13] M. Izadi, K.J. Ansari, H.M. Srivastava, *A highly accurate and efficient Genocchi-based spectral technique applied to singular fractional order boundary value problems*, Math. Methods Appl. Sci., **48**(1) (2025), 905–925.

[14] I. Khan, K.J. Ansari, R. Amin, Sundas, H. Farheen, *Application of linear Legendre multi-wavelets collocation method for solution of fourth order boundary value problems*, Eng. with Comput., **41**(6) (2024), 4097–4107.

[15] D.Y. Sucu, S.B.G. Karakoc, M. Gungor, *Investigation on the new numerical soliton solutions of FitzHugh-Nagumo equation with collocation method*, Fundam. J. Math. Appl., **8**(3) (2025), 180–186.

[16] M. Tatari, M. Dehghan, M. Razzaghi, *Application of the Adomian decomposition method for the Fokker–Planck equation*, Math. Comput. Modelling, **45**(5-6) (2007), 639–650.

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