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NEW GENERALIZATIONS FOR $s-$CONVEX FUNCTIONS VIA CONFORMABLE FRACTIONAL INTEGRALS

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Abstract. In this paper, we have obtained integral inequalities containing conformable fractional integral operators for $s-$convex functions by separating the $[a,b]$ interval to j equal sub-intervals. These inequalities are the generalizations that vary with parameter j. In this way, we give different examples of inequalities by changing this parameter.

1. Introduction

A function $f:[a,b]\to\mathbb{R}$ is said to be convex, if we have

$$f\left(\alpha x+\left(1-\alpha\right)y\right)\leq\alpha f\left(x\right)+\left(1-\alpha\right)f\left(y\right)$$

for all $x,y\in[a,b]$ and $\alpha\in[0,1]$.

Definition 1.1. [11] A function $f:\mathbb{R}_{+}\to\mathbb{R}$, where $\mathbb{R}_{+}=[0,\infty)$, is said to be $s-$convex in the first sense if

$$f(\alpha x+\beta y)\leq\alpha^{s}f(x)+\beta^{s}f(y)$$

for all $x,y\in\mathbb{R}_{+}$, $\alpha,\beta\geq0$ with $\alpha^{s}+\beta^{s}=1$ and for some fixed $s\in(0,1]$. We denote by K_{s}^{1} the class of all $s-$convex functions.

Definition 1.2. [4] A function $f:\mathbb{R}_{+}\to\mathbb{R}$, where $\mathbb{R}_{+}=[0,\infty)$, is said to be $s-$convex in the second sense if

$$f(\alpha x+\beta y)\leq\alpha^{s}f(x)+\beta^{s}f(y)$$

for all $x,y\in\mathbb{R}_{+}$, $\alpha,\beta\geq0$ with $\alpha+\beta=1$ and for some fixed $s\in(0,1]$. We denote by K_{s}^{2} the class of all $s-$convex functions.

If we choose $s=1$, both definitions reduced to ordinary concept of convexity.

A motivating inequality of Hadamard type has been proved by Latif and Dragomir in [9] as following:

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Theorem 1.1. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function on I° such that $f' \in L[a,b]$, where $a, b \in I$ with $a < b$. If $|f'|$ is convex on $[a,b]$ then the following inequality holds:*

$$\left| \frac{f\left(\frac{3a+b}{4}\right) + f\left(\frac{a+3b}{4}\right)}{2} - \frac{1}{b-a} \int_a^b f(x)\,dx \right| \le \left(\frac{b-a}{96}\right) \left[|f'(a)| + 4\left|f'\left(\frac{3a+b}{4}\right)\right| + 2\left|f'\left(\frac{a+b}{2}\right)\right| + 4\left|f'\left(\frac{a+3b}{4}\right)\right| + |f'(b)| \right].$$

In [12], Özdemir et al. presented the following generalization:

Theorem 1.2. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function on I° where $a, b \in I$ with $a < b$. If $|f'|$ is convex on $[a,b]$ then the following inequality holds:*

$$\left| \sum_{k=0}^{\frac{n-1}{2}} 2f\left(\frac{a(n-2k)+b(2k+1)}{n+1}\right) - \frac{n+1}{b-a}\int_a^b f(x)\,dx \right|$$
$$\le \frac{b-a}{6(n+1)} \sum_{k=0}^{\frac{n-1}{2}} \left(4\left|f'\left(\frac{a(n-2k)+b(2k+1)}{n+1}\right)\right| + \left|f'\left(\frac{a(n-2k+1)+b(2k)}{n+1}\right)\right| + \left|f'\left(\frac{a(n-2k-1)+b(2k+2)}{n+1}\right)\right| \right)$$

where n is an odd number.

In [8], Khalil et al. gave a new definition that is called "conformable fractional derivative". They not only proved further properties of this definitions but also gave the differences with the other fractional derivatives. Besides, another considerable study have presented by Abdeljawad to discuss the basic concepts of fractional calculus. Scientists stated that these definitions of this new fractional derivative and integral are an understandable, feasible and effective definitions. In [1], Abdeljawad gave the following definitions of right-left conformable fractional integrals:

Definition 1.3. Let $\alpha \in (n, n+1]$, $n = 0, 1, 2, ...$ and set $\beta = \alpha - n$. Then the left and right conformable fractional integral of any order $\alpha > 0$ is defined by respectively

$$\left(I_\alpha^a f\right)(t) = \frac{1}{n!}\int_a^t (t-x)^n (x-a)^{\beta-1} f(x)\,dx,$$

and

$$\left({}^b I_\alpha f\right)(t) = \frac{1}{n!}\int_t^b (x-t)^n (b-x)^{\beta-1} f(x)\,dx.$$

Let us recall the Beta function defined as follows:

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt, \qquad a, b > 0,$$

where $\Gamma(\alpha)$ is Gamma function. The incomplete Beta function is defined by

$$B_x(a,b) = \int_0^x t^{a-1}(1-t)^{b-1}\,dt.$$

Based on the above definition, Set and Çelik presented the following identity in [14]:

Lemma 1.1. *Assume that $a,b \in \mathbb{R}$ with $a<b$ and $f:[a,b]\to\mathbb{R}$ is a differentiable function on (a,b). If $f' \in L[a,b]$ then the following equality holds:*

$$\begin{aligned}
&\Psi_\alpha(a,b)\\
=\ &\frac{-(b-a)\alpha}{16}\left[\int_0^1 B_t(n+1,\alpha-n)\,f'\left(ta+(1-t)\frac{3a+b}{4}\right)dt\right.\\
&-\int_0^1 B_{1-t}(\alpha-n,n+1)\,f'\left(t\frac{3a+b}{4}+(1-t)\frac{a+b}{2}\right)dt\\
&+\int_0^1 B_t(n+1,\alpha-n)\,f'\left(t\frac{a+b}{2}+(1-t)\frac{a+3b}{4}\right)dt\\
&\left.-\int_0^1 B_{1-t}(\alpha-n,n+1)\,f'\left(t\frac{a+3b}{4}+(1-t)b\right)dt\right]
\end{aligned}$$

for $\alpha \in (n,n+1]$, $n=0,1,2,...$ where $B_t(.,.)$ is incompleted beta function and

$$\begin{aligned}
&\Psi_\alpha(a,b)\\
=\ &\frac{\alpha}{4}\left[B(n+1,\alpha-n)\left(f(a)+f\left(\frac{a+b}{2}\right)\right)\right.\\
&\left.+B(\alpha-n,n+1)\left(f\left(\frac{a+b}{2}\right)+f(b)\right)\right]-\frac{\alpha 4^{\alpha-1}n!}{(b-a)^\alpha}\\
&\times\left[\left((I_\alpha^a f)\left(\frac{3a+b}{4}\right)+\left(I_\alpha^{\frac{3a+b}{4}}f\right)\left(\frac{a+b}{2}\right)+\left(I_\alpha^{\frac{a+b}{2}}f\right)\left(\frac{a+3b}{4}\right)+\left(I_\alpha^{\frac{a+3b}{4}}f\right)(b)\right)\right].
\end{aligned}$$

For the recent studies of inequalities including conformable fractional integrals, we can refer the papers [2,3,6,10,13,15–19].

The main aim of this paper is to prove a generalization of Lemma 1 and establish some more general integral inequalities for convex functions by using conformable fractional integral operators.

2. Main Results

In order to prove the main results, we need the following integral identity that involve conformable fractional integral operator.

Lemma 2.1. [7] *Let $f:[a,b]\rightarrow\mathbb{R}$ is a differentiable mapping on (a,b) where $a,b\in\mathbb{R}$ with $a<b$. If $f'\in L[a,b]$, then the following identity holds:*

$$\begin{aligned}&\sum_{k=0}^{j-1}\int_0^1\left[B_t(n+1,\alpha-n)f'\left[t\lambda(k+1)+(1-t)\lambda(k)\right]\right]dt\\ =&\frac{j}{b-a}\sum_{k=0}^{j-1}\left\{B(n+1,\alpha-n)f\left[\lambda(k+1)\right]-n!\left(\frac{j}{b-a}\right)^{\alpha}\left({}^{\lambda(k+1)}I_{\alpha}f\right)(\lambda(k))\right\}\end{aligned}$$

for $\alpha\epsilon(n,n+1]$, $n=0,1,2,..$where $j\in\mathbb{Z}^{+}$ and for $k\in\mathbb{Z}$, $\lambda(k)=\frac{k}{j}(b-a)+a$.

Theorem 2.1. *Let $f:I\subset\mathbb{R}\rightarrow\mathbb{R}$ be differentiable on I° such that $f'\in L[a,b]$ with $a,b\in I$, $a<b$ and $\alpha>0$. If $|f'|$ is $s-$convex on $[a,b]$ in the second sense with $s\in(0,1]$, then the following inequality holds for conformable fractional integrals:*

$$\begin{aligned}&\left|\frac{j}{b-a}\sum_{k=0}^{j-1}\left\{B(n+1,\alpha-n)f\left[\lambda(k+1)\right]-n!\left(\frac{j}{b-a}\right)^{\alpha}\left({}^{\lambda(k+1)}I_{\alpha}f\right)(\lambda(k))\right\}\right|\\ \leq&\sum_{k=0}^{j-1}\left\{\frac{B(n+1,\alpha-n)-B(n+s+2,\alpha-n)}{s+1}\left|f'(\lambda(k+1))\right|\right.\\ &\left.+\frac{B(n+1,\alpha-n+s+1)}{s+1}\left|f'(\lambda(k))\right|\right\},\end{aligned}$$

where $j\in\mathbb{Z}^{+}$, $\alpha\epsilon(n,n+1]$, $n=0,1,2,...$ and for $k\in\mathbb{Z}$, $\lambda(k)=\frac{k}{j}(b-a)+a$.

Proof. Using Lemma 2.1 and triangle inequality, we can write

$$\begin{aligned}&\left|\frac{j}{b-a}\sum_{k=0}^{j-1}\left\{B(n+1,\alpha-n)f\left[\lambda(k+1)\right]-n!\left(\frac{j}{b-a}\right)^{\alpha}\left({}^{\lambda(k+1)}I_{\alpha}f\right)(\lambda(k))\right\}\right|\\ \leq&\sum_{k=0}^{j-1}\int_0^1 B_t(n+1,\alpha-n)\left|f'\left[t\lambda(k+1)+(1-t)\lambda(k)\right]\right|dt.\end{aligned}$$

Since $|f'|$ is second sense $s-$convex, then we have

$$\begin{aligned}&\int_0^1\left[B_t(n+1,\alpha-n)\left|f'\left[t\lambda(k+1)+(1-t)\lambda(k)\right]\right|\right]dt\\ \leq&\int_0^1 B_t(n+1,\alpha-n)\left[t^s\left|f'(\lambda(k+1))\right|+(1-t)^s\left|f'(\lambda(k))\right|\right]dt\end{aligned}$$

Using the properties of Beta function and integrating by parts, we obtain;

$$\begin{aligned}\int_0^1 B_t(n+1,\alpha-n)t^s dt&=B_t(n+1,\alpha-n)\left.\frac{t^{s+1}}{s+1}\right|_0^1-\int_0^1 t^n(1-t)^{\alpha-n-1}\frac{t^{s+1}}{s+1}dt\\ &=\frac{B(n+1,\alpha-n)-B(n+s+2,\alpha-n)}{s+1}\end{aligned}$$

and

$$\begin{aligned}
&\int_0^1 B_t(n+1,\alpha-n)(1-t)^s\,dt \\
&= B_t(n+1,\alpha-n)\left.\frac{-(1-t)^{s+1}}{s+1}\right|_0^1-\int_0^1 t^n(1-t)^{\alpha-n-1}\frac{-(1-t)^{s+1}}{s+1}dt \\
&= \frac{B(n+1,\alpha-n+s+1)}{s+1}.
\end{aligned}$$

We get the desired result. □

Corollary 2.1. *Under the conditions of Theorem 2.1, if we choose $j=2$, we have*

$$\begin{aligned}
&\left|\frac{2}{b-a}\left\{B(n+1,\alpha-n)\left[f\left(\frac{a+b}{2}\right)+f(b)\right]\right.\right.\\
&\left.\left.-n!\left(\frac{2}{b-a}\right)^{\alpha}\left[\left({}^{\frac{a+b}{2}}I_\alpha f\right)(a)+\left({}^{b}I_\alpha f\right)\left(\frac{a+b}{2}\right)\right]\right\}\right| \\
&\leq \frac{B(n+1,\alpha-n)-B(n+s+2,\alpha-n)}{s+1}\left[\left|f'\left(\frac{a+b}{2}\right)\right|+|f'(b)|\right] \\
&+\frac{B(n+1,\alpha-n+s+1)}{s+1}\left[|f'(a)|+\left|f'\left(\frac{a+b}{2}\right)\right|\right].
\end{aligned}$$

Corollary 2.2. *In Theorem 2.1, if we set $\alpha=1$ and $n=0$, one can obtain*

$$\begin{aligned}
&\left|\frac{j}{b-a}\sum_{k=0}^{j-1}\left\{f[\lambda(k+1)]-\frac{j}{b-a}\int_{\lambda(k)}^{\lambda(k+1)}f(x)\,dx\right\}\right| \\
&\leq \sum_{k=0}^{j-1}\left\{\frac{1-B(s+2,1)}{s+1}|f'(\lambda(k+1))|+\frac{B(1,s+2)}{s+1}|f'(\lambda(k))|\right\}.
\end{aligned}$$

Theorem 2.2. *Let $f: I\subset\mathbb{R}\to\mathbb{R}$ be differentiable on I° such that $f'\in L[a,b]$ with $a,b\in I$, $a<b$ and $\alpha>0$. If $|f'|^q$ is $s-$convex on $[a,b]$ in the second sense with $s\in(0,1]$ and $q>1$ then the following inequality holds for conformable fractional integrals:*

$$\begin{aligned}
&\left|\frac{j}{b-a}\sum_{k=0}^{j-1}\left\{B(n+1,\alpha-n)f[\lambda(k+1)]-n!\left(\frac{j}{b-a}\right)^{\alpha}\left({}^{\lambda(k+1)}I_\alpha f\right)(\lambda(k))\right\}\right| \\
&\leq \left(\int_0^1|B_t(n+1,\alpha-n)|^p\,dt\right)^{\frac{1}{p}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}\sum_{k=0}^{j-1}\left\{|f'(\lambda(k+1))|^q+|f'(\lambda(k))|^q\right\}^{\frac{1}{q}},
\end{aligned}$$

where $j\in\mathbb{Z}^+$, $\frac{1}{p}+\frac{1}{q}=1, \alpha\epsilon(n,n+1]$, $n=0,1,2,...$ and for $k\in\mathbb{Z}$, $\lambda(k)=\frac{k}{j}(b-a)+a$.

Proof. By using Lemma 2.1 and Hölder inequality, we obtain

$$\left|\frac{j}{b-a}\sum_{k=0}^{j-1}\left\{B\left(n+1,\alpha-n\right)f\left[\lambda\left(k+1\right)\right]-n!\left(\frac{j}{b-a}\right)^{\alpha}\left({}^{\lambda(k+1)}I_{\alpha}f\right)\left(\lambda\left(k\right)\right)\right\}\right| \tag{2.1}$$
$$\leq \sum_{k=0}^{j-1}\int_{0}^{1}B_{t}\left(n+1,\alpha-n\right)\left|f'\left[t\lambda\left(k+1\right)+\left(1-t\right)\lambda\left(k\right)\right]\right|dt$$
$$\leq \left(\int_{0}^{1}\left|B_{t}\left(n+1,\alpha-n\right)\right|^{p}dt\right)^{\frac{1}{p}}\sum_{k=0}^{j-1}\left(\int_{0}^{1}\left|f'\left[t\lambda\left(k+1\right)+\left(1-t\right)\lambda\left(k\right)\right]\right|^{q}dt\right)^{\frac{1}{q}}.$$

Since $|f'|^q$ is second sense $s-$convex, we can write

$$\int_{0}^{1}\left|f'\left[t\lambda\left(k+1\right)+\left(1-t\right)\lambda\left(k\right)\right]\right|^{q}dt$$
$$\leq \int_{0}^{1}\left[t^{s}\left|f'\left(\lambda\left(k+1\right)\right)\right|^{q}+\left(1-t\right)^{s}\left|f'\left(\lambda\left(k\right)\right)\right|^{q}\right]dt$$
$$= \frac{1}{s+1}\left\{\left|f'\left(\lambda\left(k+1\right)\right)\right|^{q}+\left|f'\left(\lambda\left(k\right)\right)\right|^{q}\right\}.$$

Writing these results in (2.1) completes the proof. □

Corollary 2.3. *Under the conditions of Theorem 2.2, if we choose $j=2$, we have*

$$\left|\frac{2}{b-a}\left\{B\left(n+1,\alpha-n\right)\left[f\left(\frac{a+b}{2}\right)+f\left(b\right)\right]\right.\right.$$
$$\left.\left.-n!\left(\frac{2}{b-a}\right)^{\alpha}\left[\left({}^{\frac{a+b}{2}}I_{\alpha}f\right)\left(a\right)+\left({}^{b}I_{\alpha}f\right)\left(\frac{a+b}{2}\right)\right]\right\}\right|$$
$$\leq \left(\int_{0}^{1}\left|B_{t}\left(n+1,\alpha-n\right)\right|^{p}dt\right)^{\frac{1}{p}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}$$
$$\times\left\{\left[\left|f'\left(\frac{a+b}{2}\right)\right|^{q}+\left|f'\left(a\right)\right|^{q}\right]^{\frac{1}{q}}+\left[\left|f'\left(b\right)\right|^{q}+\left|f'\left(\frac{a+b}{2}\right)\right|^{q}\right]^{\frac{1}{q}}\right\}.$$

Corollary 2.4. *In Theorem 2.2, if we set $\alpha=1$ and $n=0$, we obtain the following inequality;*

$$\left|\frac{j}{b-a}\sum_{k=0}^{j-1}\left\{f\left[\lambda\left(k+1\right)\right]-\frac{j}{b-a}\int_{\lambda(k)}^{\lambda(k+1)}f\left(x\right)dx\right\}\right|$$
$$\leq \left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}\sum_{k=0}^{j-1}\left\{\left|f'\left(\lambda\left(k+1\right)\right)\right|^{q}+\left|f'\left(\lambda\left(k\right)\right)\right|^{q}\right\}^{\frac{1}{q}}.$$

Theorem 2.3. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be differentiable on I° such that $f' \in L[a,b]$ with $a, b \in I$, $a < b$ and $\alpha > 0$. If $|f'|^q$ is $s-$convex on $[a,b]$ in the second sense with $s \in (0,1]$ and $q \geq 1$ then the following inequality holds for conformable fractional integrals:*

$$\left| \frac{j}{b-a} \sum_{k=0}^{j-1} \left\{ B(n+1, \alpha-n) f[\lambda(k+1)] - n! \left(\frac{j}{b-a} \right)^\alpha \left({}^{\lambda(k+1)}I_\alpha f \right)(\lambda(k)) \right\} \right|$$
$$\leq (B(n+1, \alpha-n+1))^{1-\frac{1}{q}} \sum_{k=0}^{j-1} \left\{ \frac{B(n+1, \alpha-n) - B(n+s+2, \alpha-n)}{s+1} |f'(\lambda(k+1))|^q \right.$$
$$\left. + \frac{B(n+1, \alpha-n+s+1)}{s+1} |f'(\lambda(k))|^q \right\}^{\frac{1}{q}},$$

where $j \in \mathbb{Z}^+$, $\alpha \epsilon (n, n+1]$, $n = 0, 1, 2, \ldots$ and for $k \in \mathbb{Z}$, $\lambda(k) = \frac{k}{j}(b-a) + a$.

Proof. By using Lemma 2.1 and power mean inequality, we have

$$\left| \frac{j}{b-a} \sum_{k=0}^{j-1} \left\{ B(n+1, \alpha-n) f[\lambda(k+1)] - n! \left(\frac{j}{b-a} \right)^\alpha \left({}^{\lambda(k+1)}I_\alpha f \right)(\lambda(k)) \right\} \right|$$
$$\leq \sum_{k=0}^{j-1} \int_0^1 B_t(n+1, \alpha-n) \left| f'[t\lambda(k+1) + (1-t)\lambda(k)] \right| dt$$
$$\leq \left(\int_0^1 B_t(n+1, \alpha-n)\, dt \right)^{1-\frac{1}{q}} \sum_{k=0}^{j-1} \left(\int_0^1 B_t(n+1, \alpha-n) \left| f'[t\lambda(k+1) + (1-t)\lambda(k)] \right|^q dt \right)^{\frac{1}{q}}.$$

By using integrating by parts, we get

$$\begin{aligned} \int_0^1 B_t(n+1, \alpha-n)\, dt &= B_t(n+1, \alpha-n)\, t|_0^1 - \int_0^1 t^{n+1}(1-t)^{\alpha-n-1} dt \\ &= B(n+1, \alpha-n) - B(n+2, \alpha-n) \\ &= B(n+1, \alpha-n+1) \end{aligned}$$

Since $|f'|^q$ is $s-$convex in the second sense then we have

$$\int_0^1 B_t(n+1, \alpha-n) \left| f'[t\lambda(k+1) + (1-t)\lambda(k)] \right|^q dt$$
$$\leq \int_0^1 B_t(n+1, \alpha-n) \left[t^s |f'(\lambda(k+1))|^q + (1-t)^s |f'(\lambda(k))|^q \right] dt$$
$$= \frac{B(n+1, \alpha-n) - B(n+s+2, \alpha-n)}{s+1} |f'(\lambda(k+1))|^q + \frac{B(n+1, \alpha-n+s+1)}{s+1} |f'(\lambda(k))|^q.$$

Combining these results, the proof is completed. □

Corollary 2.5. *Under the conditions of Theorem 2.3, if we choose $j = 2$, we have*

$$
\begin{aligned}
&\left|\frac{2}{b-a}\left\{B(n+1,\alpha-n)\left[f\left(\frac{a+b}{2}\right)+f(b)\right]\right.\right.\\
&\left.\left.-n!\left(\frac{2}{b-a}\right)^{\alpha}\left[\left({}^{\frac{a+b}{2}}I_{\alpha}f\right)(a)+\left({}^{b}I_{\alpha}f\right)\left(\frac{a+b}{2}\right)\right]\right\}\right|\\
\leq\ &(B(n+1,\alpha-n+1))^{1-\frac{1}{q}}\\
&\times\left\{\left[\frac{B(n+1,\alpha-n)-B(n+s+2,\alpha-n)}{s+1}\left|f'\left(\frac{a+b}{2}\right)\right|^{q}+\frac{B(n+1,\alpha-n+s+1)}{s+1}\left|f'(a)\right|^{q}\right]^{\frac{1}{q}}\right.\\
&\left.+\left[\frac{B(n+1,\alpha-n)-B(n+s+2,\alpha-n)}{s+1}\left|f'(b)\right|^{q}+\frac{B(n+1,\alpha-n+s+1)}{s+1}\left|f'\left(\frac{a+b}{2}\right)\right|^{q}\right]^{\frac{1}{q}}\right\}.
\end{aligned}
$$

Corollary 2.6. *In Theorem 2.3, if we take $\alpha=1$ and $n=0$, we have*

$$
\begin{aligned}
&\left|\frac{j}{b-a}\sum_{k=0}^{j-1}\left\{f[\lambda(k+1)]-\frac{j}{b-a}\int_{\lambda(k)}^{\lambda(k+1)}f(x)\,dx\right\}\right|\\
\leq\ &\left(\frac{1}{2}\right)^{1-\frac{1}{q}}\sum_{k=0}^{j-1}\left\{\frac{1-B(s+2,1)}{s+1}\left|f'(\lambda(k+1))\right|^{q}+\frac{B(1,s+2)}{s+1}\left|f'(\lambda(k))\right|^{q}\right\}^{\frac{1}{q}}.
\end{aligned}
$$

3. Conclusion

In this study we give generalizations as in [7] for $s-$convex functions. Also by using these generalizations, we give some new inequalities by choosing parameter.

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References

[1] T. Abdeljawad, *On conformable fractional calculus,* Journal of Computational and Applied Mathematics, **279** (2015), 57–66.

[2] T. Abdeljawad, J. Alzabut, F. Jarad, *A generalized Lyapunov-type inequality in the frame of conformable derivatives,* Advances in Difference Equations, 2017, **321** (2017). https://doi.org/10.1186/s13662-017-1383-z

[3] A.O. Akdemir, A. Ekinci and E. Set, *Conformable fractional integrals and related new integral inequalities,* Journal of Nonlinear and Convex Analysis, **18**(4) (2017), 661–674.

[4] W.W. Breckner, *Stetigkeitsaussagen für eine Klasse verallgemeinerter konvexer funktionen in topologischen linearen Raumen*, Publ. Inst. Math, **23** (1978), 13–20.

[5] S.S. Dragomir and C.E.M. Pearce, *Selected Topic on Hermite- Hadamard Inequalities and Applications,* Melbourne and Adelaide, December 2000.

[6] A. Ekinci, M.E. Özdemir, *Some New Integral Inequalities Via Riemann-Liouville Integral Operators,* Applied and Computational Mathematics, **18**(3) (2019), 288–295.

[7] A. Ekinci, N. Eroğlu, *New Generalizations for Convex Functions via Conformable Fractional Integrals,* Filomat, **33**(14) (2019), 4525–4534.

[8] R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, *A new denition of fractional derivative,* Journal of Computational and Applied Mathematics, **264** (2014), 65–70.

[9] M.A. Latif, S.S. Dragomir, *New Inequalities of Hermite-Hadamard Type for Functions Whose Derivatives in Absolute Value are Convex with Applications,* Acta Univ. M. Belii, Ser. Math., **21** (2013), 27–42. (2013).

[10] D. Nie, S. Rashid, A.O. Akdemir, D. Baleanu, J.B. Liu, *On Some New Weighted Inequalities for Differentiable Exponentially Convex and Exponentially Quasi-Convex Functions with Applications,* Mathematics, **7**(8) (2019), 1–12.

[11] W. Orlicz, *A note on modular spaces I.,* Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys, **9** (1961), 157–162.

[12] M.E. Özdemir, A. Ekinci, A.O. Akdemir, *Some New Integral Inequalities for Functions Whose Derivatives of Absolute Values are Convex and Concave,* TWMS J. Pure Appl. Math., **10**(2) (2019), 212–214.

[13] E. Set, A.O. Akdemir, and B. Çelik, *Some Hermite-Hadamard Type Inequalities for Products of Two Different Convex Functions via Conformable Fractional Integrals,* Xth International Statistics Days Conference, 2016.

[14] E. Set, B. Çelik, *Certain Hermite-Hadamard type inequalities associated with conformable fractional integral operators,* Creative Math. Inform., **26** (2017), 321–330.

[15] E. Set, A. Gözpınar and A. Ekinci, *Hermite-Hadamard Type Inequalities via Conformable Fractional Integrals,* Acta Mathematica Universitatis Comenianae, **86**(2) (2017), 309–320.

[16] E. Set, A.O. Akdemir, M.E. Özdemir, *Simpson Type Integral Inequalities for Convex Functions via Riemann Liouville Integrals,* Filomat, 31(14) (2017), 4415–4420.

[17] E. Set, A.O. Akdemir, İ. Mumcu, *The Hermite-Hadamard's inequality and its extentions for conformable fractional integrals of any order $\alpha > 0$,* Creative Math. Inform., **27**(2) (2018), 197–206.

[18] E. Set, A.O. Akdemir, E.A. Alan, *Hermite Hadamard and Hermite Hadamard Fejer Type Inequalities Involving Fractional Integral Operators,* Filomat, **33**(8) (2019), 2367–2380.

[19] F. Usta, M.Z. Sarıkaya, *Some Improvements of Conformable Fractional Integral Inequalities,* International Journal of Analysis and Applications, **14**(2) (2017), 162–166.

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