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SOME INTEGRAL INEQUALITIES VIA $\phi_\lambda\eta$-PREINVEX FUNCTIONS

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Abstract. The paper presents Hermite-Hadamard type inequalities, which involve Riemann-Liouville fractional integrals and contain an arbitrary parameter from the interval of definition of twice differentiable convex and concave functions.

1. Introduction

A function $f : \mathcal{C} \to \mathbb{R}$ is said to be convex, if

$$f((1-t)u + tv) \leq (1-t)f(u) + tf(v), \quad \forall u, v \in \mathcal{C}, t \in [0,1]. \tag{1.1}$$

In recent decades theory of convexity has experienced rapid development resultantly attracted many researchers. Several new generalizations of classical convexity have been proposed in the literature, for example see [2]. A factor which makes theory of convexity more fascinating is its close relationship with theory of inequalities. Many results obtained in inequalities theory are direct consequences of convex functions. The result of Hermite and Hadamard reads as follows:
Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a convex function, where $a, b \in I$ with $a < b$. Then

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x \leq \frac{f(a)+f(b)}{2},$$

played a significant role in the in theory of convexity and inequalities. For some more details on Hermite-Hadamard's type of inequalities, see [1, 3–10, 12, 14].
Recently Gordji et al. [7] introduced the concept of η-convexity. Under some suitable condition the class of η-convex functions reduces to the class of class of classical convex functions. In [7] authors have also obtained a new extension of Hermite-Hadamard type inequality via η-convex functions.
Inspired by the recent research of this field, we in this paper introduce a new class of η-convex functions which is called as $\phi_\lambda\eta$-preinvex functions. We derive some new inequalities

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of Hermite-Hadamard via $\phi_\lambda\eta$-preinvex functions. Some special cases are also discussed. This is the main motivation of this paper.

2. Preliminaries

In this section, we recall some previously known concepts and basic results for η-convex functions.

Definition 2.1 ([7])**.** A function $f : I = [a, b] \to \mathbb{R}$ is said to be η-convex function, if there exists a bifunction $\eta(.,.)$, such that

$$f((1-t)u + tv) \leq f(u) + t\eta(f(v), f(u)), \quad \forall u, v \in I, t \in [0,1]. \tag{2.1}$$

Note that, if $\eta(f(v), f(u)) = f(v) - f(u)$ in the above inequality, then, we have the definition of classical convex functions.
Now we give some examples of η-convex functions which are not convex in the classical sense.

Example 2.1 ([7]). Let $f : \mathbb{R}_+ \to \mathbb{R}$ be defined as:

$$f(x) = \begin{cases} -x, & if\, x \geq 0, \\ x, & if\, x < 0, \end{cases}$$

and suppose $\eta(x, y) = -x - y$, for all $x, y \in (-\infty, 0]$.

Example 2.2 ([7]). Let $f : \mathbb{R} \to \mathbb{R}_+$ be defined as:

$$f(x) = \begin{cases} x, & if\, 0 \leq x \leq 1, \\ 1, & if\, x > 1, \end{cases}$$

and suppose

$$\eta(x, y) = \begin{cases} x + y, & if\, x \leq y, \\ 2(x + y), & if\, x > y, \end{cases}$$

for all $x, y \in [0, +\infty)$.

For some useful details on η-convex functions, see [7].

Definition 2.2 ([13])**.** A set $\mathcal{K}_\phi$ is said to be ϕ_λ-invex with respect to $\phi(.) : \mathbb{R} \to \mathbb{R}$ and bifunction $\lambda(.,.) : \mathcal{K}_\phi \times \mathcal{K}_\phi \to \mathbb{R}^n$, if

$$u + te^{i\phi}\lambda(v, u) \in K_\phi, \quad \forall u, v \in K_\phi, t \in [0, 1].$$

Note that if $\lambda(v, u) = v - u$ we have the class of ϕ-convex set. And if along with $\lambda(v, u) = v - u$ we have $\phi = 0$ we have classical convex set.
We now define the class of $\phi_\lambda\eta$-preinvex functions.

Definition 2.3. A function f on ϕ_λ-invex set is said to be $\phi_\lambda\eta$-preinvex function, if there exists bifunctions $\lambda(.,.) : \mathcal{K}_\phi \times \mathcal{K}_\phi \to \mathbb{R}^n$, $\eta(.,.) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, such that

$$f(u + te^{i\phi}\lambda(v, u)) \leq f(u) + t\eta(f(v), f(u)), \quad \forall u, v \in K_\phi, t \in [0, 1]. \tag{2.2}$$

Note that if we take $\eta(f(v), f(u)) = f(v) - f(u)$ we get the definition of ϕ_λ-preinvex functions defined by Noor [13]. If $\lambda(v,u) = v - u$ we get the definition of $\phi\eta$-convex function. If $\lambda(v,u) = v - u$ and $\phi = 0$ we get η-convex function [7]. If $\lambda(v,u) = v - u$, $\eta(f(v), f(u)) = f(v) - f(u)$ and $\phi = 0$ we get the classical convex functions. If we take $\eta(f(v), f(u)) = f(v) - f(u)$ and $\phi = 0$ we get the definition of classical preinvex functions [16].

Definition 2.4. A function f on ϕ_λ-invex set is said to be $\phi_\lambda\eta$-pre-quasiinvex function, if there exists bifunctions $\lambda(.,.) : \mathcal{K}_\phi \times \mathcal{K}_\phi \to \mathbb{R}^n$ and $\eta(.,.) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, such that

$$f(u + te^{i\phi}\lambda(v,u)) \leq \max\{f(u), f(u) + \eta(f(v), f(u))\}, \quad \forall u, v \in K_\phi, t \in [0,1].$$

The following results will be used in obtaining our main results.

Lemma 2.1 ([14]). *Let $f : \mathcal{I}_\phi = [a, a + e^{i\phi}\lambda(b,a)] \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function, where $a < a + e^{i\phi}\lambda(b,a)$ and $0 \leq \phi \leq \frac{\pi}{2}$. If $f' \in L_1[a, a + e^{i\phi}\lambda(b,a)]$, then*

$$\begin{aligned} R_{f'}(a,b) &= -\frac{f(a) + f(a + e^{i\phi}\lambda(b,a))}{2} + \frac{1}{e^{i\phi}\lambda(b,a)} \int_a^{a+e^{i\phi}\lambda(b,a)} f(x)\mathrm{d}x \\ &= \frac{e^{i\phi}\lambda(b,a)}{2} \int_0^1 (1-2t) f'(a + e^{i\phi}\lambda(b,a))\mathrm{d}t. \end{aligned}$$

Lemma 2.2 ([14]). *Let $f : \mathcal{I}_\phi = [a, a + e^{i\phi}\lambda(b,a)] \subset \mathbb{R} \to \mathbb{R}$ be a twice differentiable function, where $a < a + e^{i\phi}\lambda(b,a)$ and $0 \leq \phi \leq \frac{\pi}{2}$. If $f'' \in L_1[a, a + e^{i\phi}\lambda(b,a)]$, then*

$$\begin{aligned} R_{f''}(a,b) &= \frac{f(a) + f(a + e^{i\phi}\lambda(b,a))}{2} - \frac{1}{e^{i\phi}\lambda(b,a)} \int_a^{a+e^{i\phi}\lambda(b,a)} f(x)\mathrm{d}x \\ &= \frac{[e^{i\phi}\lambda(b,a)]^2}{2} \int_0^1 t(1-t) f''(a + e^{i\phi}\lambda(b,a))\mathrm{d}t. \end{aligned}$$

3. Main Results

In this section, we establish our main results.

Theorem 3.1. *Let $f : \mathcal{I}_\phi = [a, a + e^{i\phi}\lambda(b,a)] \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function, where $a < a + e^{i\phi}\lambda(b,a)$, $f' \in L_1[a, a + e^{i\phi}\lambda(b,a)]$ and $0 \leq \phi \leq \frac{\pi}{2}$. If $|f'|$ is $\phi_\lambda\eta$-preinvex function, then*

$$|R_{f'}(a,b)| \leq \frac{e^{i\phi}\lambda(b,a)}{8} \left\{2|f'(a)| + \eta(|f'(b)|, |f'(a)|)\right\}.$$

Proof. Using Lemma 2.1, the property of modulus and the fact that $|f'|$ is $\phi_\lambda\eta$-preinvex function, we have

$$\begin{aligned}
|R_{f'}(a,b)| &= \left|\frac{e^{i\phi}\lambda(b,a)}{2}\int_0^1(1-2t)f'(a+te^{i\phi}\lambda(b,a))\mathrm{d}t\right| \\
&\leq \frac{e^{i\phi}\lambda(b,a)}{2}\left\{\int_0^1|1-2t||f'(a+te^{i\phi}\lambda(b,a))|\right\} \\
&\leq \frac{e^{i\phi}\lambda(b,a)}{2}\left\{\int_0^1|1-2t|\left\{|f'(a)|+t\eta(|f'(b)|,|f'(a)|)\right\}\mathrm{d}t\right\}, \\
&= \frac{e^{i\phi}\lambda(b,a)}{8}\left\{2|f'(a)|+\eta(|f'(b)|,|f'(a)|)\right\}.
\end{aligned}$$

This completes the proof. □

Theorem 3.2. *Let $f : \mathfrak{I}_\phi = [a, a+e^{i\phi}\lambda(b,a)] \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function, where $a < a+e^{i\phi}\lambda(b,a)$, $f' \in L_1[a, a+e^{i\phi}\lambda(b,a)]$ and $0 \leq \phi \leq \frac{\pi}{2}$. If $|f'|^p$ is $\phi_\lambda\eta$-convex function, then*

$$|R_{f'}(a,b)| \leq \frac{e^{i\phi}\lambda(b,a)}{2^{1+\frac{1}{q}}(p+1)^{\frac{1}{p}}}\left\{(2|f'(a)|^q+\eta(|f'(b)|^q,|f'(a)|^q))\right\}^{\frac{1}{q}}.$$

Proof. Using Lemma 2.1, the property of modulus, Holder's inequality and the fact that $|f'|^p$ where $\frac{1}{p}+\frac{1}{q}=1$ is $\phi_\lambda\eta$-preinvex function, we have

$$\begin{aligned}
|R_{f'}(a,b)| &= \left|\frac{e^{i\phi}\lambda(b,a)}{2}\int_0^1(1-2t)f'(a+te^{i\phi}\lambda(b,a))\mathrm{d}t\right| \\
&\leq \frac{e^{i\phi}\lambda(b,a)}{2}\left\{\int_0^1|1-2t||f'(a+te^{i\phi}\lambda(b,a))|\mathrm{d}t\right\} \\
&\leq \frac{e^{i\phi}\lambda(b,a)}{2}\left(\int_0^1|1-2t|^p\mathrm{d}t\right)^{\frac{1}{p}}\left(\int_0^1|f'(a+te^{i\phi}\lambda(b,a))|^q\mathrm{d}t\right)^{\frac{1}{q}} \\
&\leq \frac{e^{i\phi}\lambda(b,a)}{2(p+1)^{\frac{1}{p}}}\left(\int_0^1\left[|f'(a)|^q+t\eta(|f'(b)|^q,|f'(a)|^q)\right]\mathrm{d}t\right)^{\frac{1}{q}} \\
&= \frac{e^{i\phi}\lambda(b,a)}{2^{1+\frac{1}{q}}(p+1)^{\frac{1}{p}}}\left\{(2|f'(a)|^q+\eta(|f'(b)|^q,|f'(a)|^q))\right\}^{\frac{1}{q}}.
\end{aligned}$$

This completes the proof. □

Theorem 3.3. *Let $f : \mathfrak{I}_\phi = [a, a+e^{i\phi}\lambda(b,a)] \subset \mathbb{R} \to \mathbb{R}$ be a differentiable function, where $a < a+e^{i\phi}\lambda(b,a)$, $f' \in L_1[a, a+e^{i\phi}\lambda(b,a)]$ and $0 \leq \phi \leq \frac{\pi}{2}$. If $|f'|^q$ where $q > 1$ is*

$\phi_\lambda\eta$-preinvex function, then

$$|R_{f'}(a,b)| \leq \frac{e^{i\phi}\lambda(b,a)}{2}\left(\frac{1}{2}\right)^{1+\frac{1}{q}}\left\{2|f'(a)|^{\frac{p}{p-1}}+\eta(|f'(b)|^{\frac{p}{p-1}},|f'(a)|^{\frac{p}{p-1}})\right\}^{\frac{p-1}{p}}.$$

Proof. Using Lemma 2.1, the property of modulus, power means inequality and the fact that $|f'|^q$ is $\phi_\lambda\eta$-convex function, we have

$$\begin{aligned}
|R_{f'}(a,b)| &= \left|\frac{e^{i\phi}\lambda(b,a)}{2}\int_0^1(1-2t)f'(a+te^{i\phi}\lambda(b,a))\mathrm{d}t\right| \\
&\leq \frac{e^{i\phi}\lambda(b,a)}{2}\left\{\int_0^1|1-2t||f'(a+te^{i\phi}\lambda(b,a))|\mathrm{d}t\right\} \\
&\leq \frac{e^{i\phi}\lambda(b,a)}{2}\left\{\left(\int_0^1|1-2t|\mathrm{d}t\right)^{1-\frac{1}{q}}\left(\int_0^1|1-2t||f'(a+te^{i\phi}\lambda(b,a))|^q\mathrm{d}t\right)^{\frac{1}{q}}\right\} \\
&\leq \frac{e^{i\phi}\lambda(b,a)}{2}\left(\frac{1}{2}\right)^{1-\frac{1}{q}}\left\{\int_0^1|1-2t|\left(|f'(a)|^{\frac{p}{p-1}}+t\eta(|f'(b)|^{\frac{p}{p-1}},|f'(a)|^{\frac{p}{p-1}})\right)\mathrm{d}t\right\}^{\frac{p-1}{p}} \\
&= \frac{e^{i\phi}\lambda(b,a)}{2}\left(\frac{1}{2}\right)^{1+\frac{1}{q}}\left\{2|f'(a)|^{\frac{p}{p-1}}+\eta(|f'(b)|^{\frac{p}{p-1}},|f'(a)|^{\frac{p}{p-1}})\right\}^{\frac{p-1}{p}}.
\end{aligned}$$

This completes the proof. □

Theorem 3.4. *Let $f : \mathfrak{I}_\phi = [a, a+e^{i\phi}\lambda(b,a)] \subset \mathbb{R} \to \mathbb{R}$ be a twice differentiable function, where $a < a+e^{i\phi}\lambda(b,a)$, $f'' \in L_1[a, a+e^{i\phi}\lambda(b,a)]$ and $0 \leq \phi \leq \frac{\pi}{2}$. If $|f''|$ is $\phi_\lambda\eta$-convex function, then*

$$|R_{f''}(a,b)| \leq \frac{[e^{i\phi}\lambda(b,a)]^2}{24}\left[2|f''(a)|+\eta(|f''(b)|,|f''(b)|)\right].$$

Proof. Using Lemma 2.2, the property of modulus and the fact that $|f''|$ is $\phi_\lambda\eta$-preinvex function, we have

$$\begin{aligned}
|R_{f''}(a,b)| &= \left|\frac{[e^{i\phi}\lambda(b,a)]^2}{2}\int_0^1 t(1-t)f''(a+e^{i\phi}\lambda(b,a))\mathrm{d}t\right| \\
&\leq \frac{[e^{i\phi}\lambda(b,a)]^2}{2}\left[\int_0^1 t(1-t)|f''(a+e^{i\phi}\lambda(b,a))|\mathrm{d}t\right] \\
&\leq \frac{[e^{i\phi}\lambda(b,a)]^2}{2}\left[\int_0^1 t(1-t)\{|f''(a)|+t\eta(|f''(b)|,|f''(b)|)\}\mathrm{d}t\right] \\
&= \frac{[e^{i\phi}\lambda(b,a)]^2}{24}\left[2|f''(a)|+\eta(|f''(b)|,|f''(b)|)\right].
\end{aligned}$$

This completes the proof. □

Theorem 3.5. *Let $f : \mathfrak{I}_\phi = [a, a + e^{i\phi}\lambda(b,a)] \subset \mathbb{R} \to \mathbb{R}$ be a twice differentiable function, where $a < a + e^{i\phi}\lambda(b,a)$, $f'' \in L_1[a, a + e^{i\phi}\lambda(b,a)]$ and $0 \le \phi \le \frac{\pi}{2}$. If $|f''|^p$ where $\frac{1}{p} + \frac{1}{q} = 1$ is $\phi_\lambda\eta$-preinvex function, then*

$$\begin{aligned}
&\left|R_{f''}(a,b)\right| \\
&\le \frac{[e^{i\phi}\lambda(b,a)]^2}{8}\left(\frac{\sqrt{\pi}}{2}\right)^{\frac{1}{p}}\left(\frac{\Gamma(1+p)}{\Gamma\left(\frac{3}{2}+p\right)}\right)^{\frac{1}{p}}\left(\frac{2|f''(a)|^{\frac{p}{p-1}} + \eta(|f''(b)|^{\frac{p}{p-1}}, |f''(a)|^{\frac{p}{p-1}})}{2}\right)^{\frac{p-1}{p}}.
\end{aligned}$$

Proof. Using Lemma 2.2, the property of modulus, Holder's inequality and the fact that $|f''|^p$ is $\phi_\lambda\eta$-preinvex function, we have

$$\begin{aligned}
&\left|R_{f''}(a,b)\right| \\
&= \left|\frac{[e^{i\phi}\lambda(b,a)]^2}{2}\int_0^1 t(1-t)f''(a + e^{i\phi}\lambda(b,a))\mathrm{d}t\right| \\
&\le \frac{[e^{i\phi}\lambda(b,a)]^2}{2}\left[\int_0^1 t(1-t)|f''(a + e^{i\phi}\lambda(b,a))|\mathrm{d}t\right] \\
&\le \frac{[e^{i\phi}\lambda(b,a)]^2}{2}\left(\int_0^1 (t-t^2)^p dt\right)^{\frac{1}{p}}\left(\int_0^1 |f''(a + te^{i\phi}(b-a))|^q \mathrm{d}t\right)^{\frac{1}{q}} \\
&\le \frac{[e^{i\phi}\lambda(b,a)]^2}{2}\left(\frac{2^{-1-2p}\sqrt{\pi}\Gamma(1+p)}{\Gamma\left(\frac{3}{2}+p\right)}\right)^{\frac{1}{p}}\left(\int_0^1 \{|f''(a)|^q + t(|f''(b)|^q, |f''(a)|^q)\}\mathrm{d}t\right)^{\frac{1}{q}} \\
&= \frac{[e^{i\phi}\lambda(b,a)]^2}{8}\left(\frac{\sqrt{\pi}}{2}\right)^{\frac{1}{p}}\left(\frac{\Gamma(1+p)}{\Gamma\left(\frac{3}{2}+p\right)}\right)^{\frac{1}{p}}\left(\frac{2|f''(a)|^{\frac{p}{p-1}} + \eta(|f''(b)|^{\frac{p}{p-1}}, |f''(a)|^{\frac{p}{p-1}})}{2}\right)^{\frac{p-1}{p}}.
\end{aligned}$$

This completes the proof. □

Theorem 3.6. *Let $f : \mathfrak{I}_\phi = [a, a + e^{i\phi}\lambda(b,a)] \subset \mathbb{R} \to \mathbb{R}$ be a twice differentiable function, where $a < a + e^{i\phi}\lambda(b,a)$, $f'' \in L_1[a, a + e^{i\phi}\lambda(b,a)]$ and $0 \le \phi \le \frac{\pi}{2}$. If $|f''|^q$ where $q \ge 1$ is $\phi_\lambda\eta$-preinvex function, then*

$$\left|R_{f''}(a,b)\right| \le \frac{[e^{i\phi}\lambda(b,a)]^2}{12}(6)^{\frac{p-1}{p}}\left(\frac{2|f''(a)|^{\frac{p}{p-1}} + \eta(|f''(b)|^{\frac{p}{p-1}}, |f''(a)|^{\frac{p}{p-1}})}{12}\right)^{\frac{p-1}{p}}.$$

Proof. Using Lemma 2.2, the property of modulus, power means inequality and the fact that $|f''|^q$ is $\phi_\lambda\eta$-convex function, we have

$$\begin{aligned}
|R_{f''}(a,b)| &= \left|\frac{[e^{i\phi}\lambda(b,a)]^2}{2}\int_0^1 t(1-t)f''(a+e^{i\phi}\lambda(b,a))\mathrm{d}t\right| \\
&\leq \frac{[e^{i\phi}\lambda(b,a)]^2}{2}\left[\int_0^1 t(1-t)|f''(a+e^{i\phi}\lambda(b,a))|\mathrm{d}t\right] \\
&\leq \frac{[e^{i\phi}\lambda(b,a)]^2}{2}\left(\int_0^1 (t-t^2)\mathrm{d}t\right)^{1-\frac{1}{q}}\left(\int_0^1 t(1-t)|f''(a+te^{i\phi}\lambda(b,a))|^q\mathrm{d}t\right)^{\frac{1}{q}} \\
&\leq \frac{[e^{i\phi}\lambda(b,a)]^2}{2}\left(\frac{1}{6}\right)^{1-\frac{1}{q}}\left(\int_0^1 t(1-t)\{|f''(a)|^q + t\eta(|f''(b)|^q,|f''(a)|^q)\}\mathrm{d}t\right)^{\frac{1}{q}} \\
&= \frac{[e^{i\phi}\lambda(b,a)]^2}{12}(6)^{\frac{p-1}{p}}\left(\frac{2|f''(a)|^{\frac{p}{p-1}} + \eta(|f''(b)|^{\frac{p}{p-1}},|f''(a)|^{\frac{p}{p-1}})}{12}\right)^{\frac{p-1}{p}}.
\end{aligned}$$

This completes the proof. □

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